The Math Department at Quarry Lane School has a challenging and enriching curriculum. We want to ensure each student is well prepared for the following school year. It's important for our students to keep up with their math skill over for the long summer break. Studies indicate that students lose a huge percentage of what they learned from the prior school year. In order to be proactive, Quarry Lane School Math Department would like to provide you with the following math grade level supplements.

These packets are for your student to practice during the long summer break. Each packet contains practice worksheets. Your student can do the practice worksheets at their own pace. Please encourage your student to complete this work and grade it using the answer keys provided.

Have a safe and fun-filled summer!

Name:_____

Precalculus Summer Packet

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This packet is to help you review various topics that are considered to be prerequisite knowledge upon entering PreCalculus.

- *Show all of your work NEATLY and organized!*
- *Only select answers are given on the back for reference, but all questions will be graded for correctness.*
- *You may check solutions on your calculator, but be sure to show all work for credit.*
- *Questions with NO work will receive NO credit!!*
- *Box your answers!*
- *NO CALCULATOR UNLESS OTHERWISE STATED!*

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I. Geometry Topics

- Midpoint formula: $\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$

- Median of a Δ: A segment from a vertex to the midpoint of the opposite side.

- Angle Bisector of a Δ: A segment from a vertex which bisects the angle.

- Perpendicular Bisector: A line passing through the midpoint of and perpendicular to a segment.

- Altitude of a Δ: A segment from a vertex perpendicular to the opposite side.

- Equations of a line:
  1. Slope intercept: $y = mx + b$, where slope $= \frac{y_1 - y_2}{x_1 - x_2}$
  2. Point slope: $y - y_1 = m(x - x_1)$
  3. Standard: $Ax + By = C$

- Distance formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Directions - State all linear equations in Standard Form unless otherwise stated.

1. Given ΔABC with A(-5, 4), B(1, 6) and C(3, 8), write the equation of the median from point C.

2. Write the equation of the line parallel to the line $4x - 6y = -1$ and contains the x-intercept of $3x - 2y = 12$.

3. Write the equation of the line, in slope intercept form, through (2, -4) and perpendicular to $x - 2y = 7$.

4. Find the value of "a" if a line containing the point (a, -3a) has a y-intercept of 7 and a slope of $-\frac{2}{3}$.

5. Given the distance between (x, 1) and (-2, 5) is $2\sqrt{7}$. Find the value(s) of x. Leave your answer in simplest form.

6. Write the equation of the perpendicular bisector of the segment joining A(-5, 4) and B(3, -6).

II. Quadratics

A. Factoring - Strategies to try when Factoring:

- GCF
- Difference of two squares
- Sum / Difference of cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

- Guess and Check
- Grouping

1. *Directions - Factor completely each of the following:*

a. $4x^2 + 27x + 35$

b. $-28y^2 + 7t^2$

c. $x^3 - 2x^2 - 9x + 18$

d. $8a^4 + 27ab^3$

B. Equations - Since the following are equations, we can now go a step further and solve for x by factoring or using the quadratic formula.

2. *Directions - Solve each of the following:*

a. $-3x^2 - 5x + 12 = 0$

b. $3x^2 + 5x = 6$

c. $x^2 + 2x + 3 = 0$

C. Graphing - To graph a quadratic equation in standard form, $y = ax^2 + bx + c$, find the important points of the graph by following the steps:

Y-intercept: If a point is the y-intercept of the curve, then that is the point at which the graph crosses the y-axis. Since this point is on the y-axis, then the x-coordinate must be 0. Substitute zero in for x and solve for y.

Vertex: x-coordinate of the vertex: $x = \frac{-b}{2a}$.

y-coordinate of the vertex: substitute the value found for the x-coor. into the original equation and solve for y.

X-intercepts: If a point is an x-intercept of the curve, then it is a point at which the graph crosses the x-axis. Since these points are on the x-axis, then the y-coordinates must be 0. Substitute zero in for y and solve for x by factoring or using the quadratic formula.

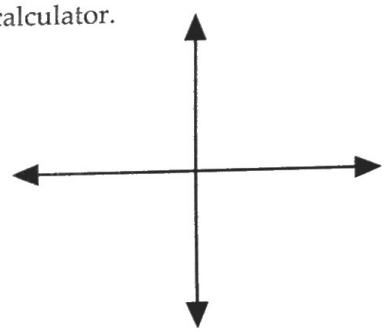
*No calculator, but you should also be able to graph with the use of your calculator.

3. *Directions - Given* $y = -3x^2 - 6x + 2$, *find and graph.*

a. y-intercept

b. vertex

c. x-intercepts



III. Systems

Substitution or Linear Combination (Elimination) can be used to solve systems of equations.

- If there is a solution to the system, then the equations are representing intersecting lines.
- If both variables cancel out and an equation is formed that is never true, then there is no solution and the lines never intersect. Lines that never intersect are parallel lines.
- If both variables cancel out and an equation is formed that is always true, then there are infinitely many solutions and the equations must represent the same line.

Directions - Solve each of the following.
- Explain what the solution tells us about the lines represented by the equations.
- No calculator, but you need to be able to solve with the use of a calculator as well.

1. $\begin{cases} 3x - 4y = 2 \\ -x + 3y = 1 \end{cases}$

2. $\begin{cases} -x + y = 3 \\ 2x - 2y = -6 \end{cases}$

Solution:_____

Explanation:

Solution:_____

Explanation:

IV. Exponents

Directions - Simplify using only positive exponents and no calculator!!!

Properties: $a^m \cdot a^n = a^{m+n}$ $\quad (a^m)^n = a^{m \cdot n}$ $\quad a^{\frac{p}{r}} = \sqrt[r]{a^p}$

$a^{-n} = \frac{1}{a^n}$ $\quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\quad \frac{a^m}{a^n} = a^{m-n}$

1. $\left(\frac{81}{64}\right)^{-\frac{1}{2}}$

2. $\left(27^{-2}\right)^{-\frac{1}{3}}$

3. $\frac{(3x^2)^{-1}}{6x^{-3}}$

4. a. -2^4

   b. $(-2)^4$

5. $\frac{3^{-5} \quad 3^{10}}{3^2}$

6. $\left(4^{-1} + 2^{-1}\right)^2$ - hint 1: $\left(a^{-m} + a^{-n}\right)^p \quad a^{-mp} + a^{-np}$

   - hint 2: Apply the neg. exponent property to each term. Then get a common denom. and add

V. Logarithms

Given $\log_b a = x$, then $b^x = a$ where $b > 0$ but $b \neq 1$, and $a > 0$.

Directions: - Solve for x.

1. $3\log_2 x = 12$

2. $\log_5 125 = x$

3. $3 + 4\log_x 4 = 5$

4. $\frac{3}{2}\log_{27}(x+5) = 1$

5. $1 + \frac{4}{3}\log_{(x-3)} 4 = \frac{11}{3}$

6. $\log_{\sqrt{3}} 25^{4x-1} = 3$

VI. Rational Expressions

Directions - Simplify to a single fraction:

1. Hint: get a common denominator!

$$\frac{1}{ab} - \frac{2}{b^2}$$

2. Hint: factor and cancel!

$$\frac{x^2 + 6x + 8}{x^2 - 4}$$

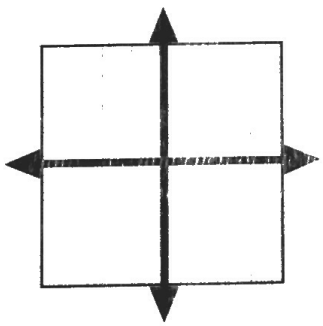
3. Hint: get a common denominator in the numerator and multiply by the reciprocal

$$\frac{\frac{x}{x-1} + 1}{\frac{x+2}{x}}$$

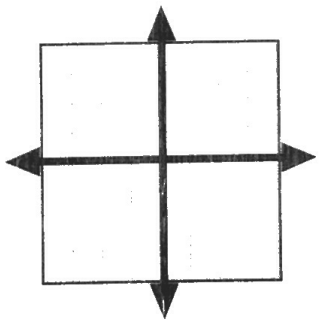
VII. Quick Graphs:

Directions - Graph each of the following.
- If you don't remember, use your graphing calculator to help you determine the patterns. But you need to be able to do these graphs without your calculator!

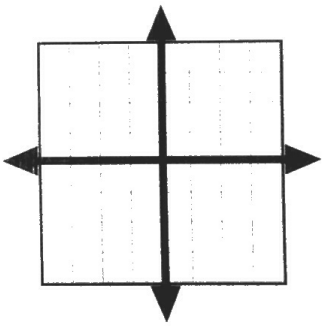
1. $y = \sqrt{x-2} - 3$



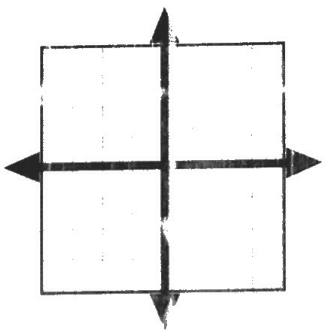
2. $y = (x+2)^2 + 1$



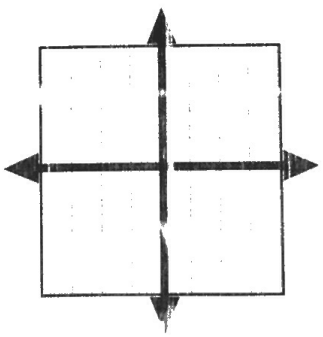
3. $y = |x| - 1$



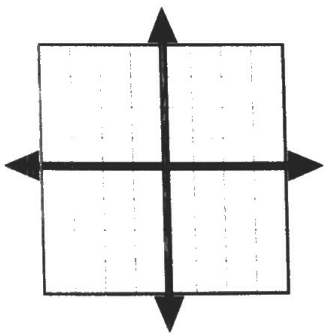
4. $y = \sqrt[3]{x+} - 2$



5. $y = (x-3)^3 + 2$



6. $y = x + 3$



VII. Probability - Calculator Allowed - *Round all probabilities to three decimal places.*

1. **Prom Plans** - A group of 10 girls and 6 boys from the sophomore class board are in charge of the 2005 prom. In how many ways can they elect a President, Vice-President and Treasurer?

2. **Drawing Cards** - Three cards are drawn *all at once* from a standard deck of 52 playing cards. What is the probability of selecting exactly two aces?

3. **Skittles in a Jar** - Stacey has a jar that contains 21 green skittles and 30 yellow skittles. She withdraws one skittle and notes the color. She does not return it to the jar. Stacey then withdraws another skittle. Find the probability that the 1st skittle is green and the next skittle is yellow.

4. **Trains** - A passenger train consists of 2 different locomotives, 6 different passenger cars and 2 different sleeper cars. In how many ways can the cars be arranged if the locomotives must be together and must be at the beginning of the train?

VIII. Regression Equations - Calculator Allowed

Regression equations are equations of lines which are created to provide functions which "best fit" a set of data points. The calculation of such equations is left up to our calculator.

In this problem, you will be asked to create both a quadratic and a linear regression equation for the data provided below. On the following page you will find directions on how to create a quadratic regression equation on the TI-83 or TI-83 Plus. Should you have a different calculator, you will have to look up this topic in the manual.

For example, in an experiment, students measured the speed s (in meters per second) of a falling object t seconds after it was released. The results are given in the table.

t	0	1	2	3	4
s	0	11	20.6	32.4	49.4

******Round all answers to three decimal places!!!******

1. Using the calculator, create a quadratic regression equation for the data.

   Quadratic Regression Equation: y= _____

2. Using the calculator, create a linear regression equation for the data.

   Linear Regression Equation: y = _____

3. Which equation best represents the data? Why do you think that?

4. Using the quadratic model for the data, what was the speed at $t = 1.5$ seconds?

5. Using the quadratic model for the data, what was the speed at $t = 6$ seconds?

6. Using the linear model for the data, when was the speed exactly 23 m/s?

Name:_____

Precalculus Summer Packet

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  1. Slope intercept: $y = mx + b$, where slope $= \frac{y_1 - y_2}{x_1 - x_2}$
  2. Point slope: $y - y_1 = m(x - x_1)$
  3. Standard: $Ax + By = C$

- Distance formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Directions - State all linear equations in Standard Form unless otherwise stated.

1. Given ΔABC with A(-5, 4), B(1, 6) and C(3, 8), write the equation of the median from point C.

$3x - 5y = -3$

2. Write the equation of the line parallel to the line $4x - 6y = -1$ and contains the x-intercept of $3x - 2y = 12$.

$2x - 3y = 8$

3. Write the equation of the line, in slope intercept form, through (2, -4) and perpendicular to $x - 2y = 7$.

$y = -2x + 8$

4. Find the value of "a" if a line containing the point (a, -3a) has a y-intercept of 7 and a slope of $-\frac{2}{3}$.

$a = -3$

5. Given the distance between (x, 1) and (-2, 5) is $2\sqrt{7}$. Find the value(s) of x. Leave your answer in simplest form.

$x = -2 \pm 2\sqrt{3}$

6. Write the equation of the perpendicular bisector of the segment joining A(-5, 4) and B(3, -6).

$4x - 5y = 1$

II. Quadratics

A. Factoring - Strategies to try when Factoring:

- GCF
- Difference of two squares
- Sum / Difference of cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

- Guess and Check
- Grouping

1. *Directions - Factor completely each of the following:*

a. $4x^2 + 27x + 35$

$(4x+7)(x+5)$

b. $-28y^2 + 7t^2$

$7(t-2y)(t+2y)$

c. $x^3 - 2x^2 - 9x + 18$

$(x-3)(x-2)(x+3)$

d. $8a^4 + 27ab^3$

$a(2a+3b)(4a^2+6ab+9b^2)$

B. Equations - Since the following are equations, we can now go a step further and solve for x by factoring or using the quadratic formula.

2. *Directions - Solve each of the following:*

a. $-3x^2 - 5x + 12 = 0$

$x = \frac{4}{3}$ or $x = -3$

b. $3x^2 + 5x = 6$

$x = \dfrac{-5 \pm \sqrt{97}}{6}$

c. $x^2 + 2x + 3 = 0$

$x = -1 \pm \sqrt{2}\,i$

C. Graphing - To graph a quadratic equation in standard form, $y = ax^2 + bx + c$, find the important points of the graph by following the steps:

Y-intercept: If a point is the y-intercept of the curve, then that is the point at which the graph crosses the y-axis. Since this point is on the y-axis, then the x-coordinate must be 0. Substitute zero in for x and solve for y.

Vertex: x-coordinate of the vertex: $x = \frac{-b}{2a}$.

y-coordinate of the vertex: substitute the value found for the x-coor. into the original equation and solve for y.

X-intercepts: If a point is an x-intercept of the curve, then it is a point at which the graph crosses the x-axis. Since these points are on the x-axis, then the y-coordinates must be 0. Substitute zero in for y and solve for x by factoring or using the quadratic formula.

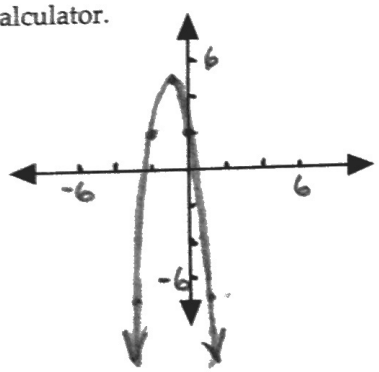
*No calculator, but you should also be able to graph with the use of your calculator.

3. *Directions - Given* $y = -3x^2 - 6x + 2$, *find and graph.*

a. y-intercept
$(0, 2)$

b. vertex
$(-1, 5)$

c. x-intercepts
$\left(-1 \pm \frac{\sqrt{15}}{3}, 0\right)$



III. Systems

Substitution or Linear Combination (Elimination) can be used to solve systems of equations.
- If there is a solution to the system, then the equations are representing intersecting lines.
- If both variables cancel out and an equation is formed that is never true, then there is no solution and the lines never intersect. Lines that never intersect are parallel lines.
- If both variables cancel out and an equation is formed that is always true, then there are infinitely many solutions and the equations must represent the same line.

Directions - Solve each of the following.
- *Explain what the solution tells us about the lines represented by the equations.*
- *No calculator, but you need to be able to solve with the use of a calculator as well.*

1. $\begin{cases} 3x - 4y = 2 \\ -x + 3y = 1 \end{cases}$

$x = 2$ & $y = 1$

The lines intersect

2. $\begin{cases} -x + y = 3 \\ 2x - 2y = -6 \end{cases}$

Infinitely many solutions

The equations represent the same line.

IV. Exponents

Directions - Simplify using only positive exponents and no calculator!!!

Properties: $a^m \cdot a^n = a^{m+n}$ $\quad (a^m)^n = a^{m \cdot n}$ $\quad a^{\frac{p}{r}} = \sqrt[r]{a^p}$

$a^{-n} = \frac{1}{a^n}$ $\quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\quad \frac{a^m}{a^n} = a^{m-n}$

1. $\left(\frac{81}{64}\right)^{-\frac{1}{2}}$ $\frac{8}{9}$

2. $\left(27^{-2}\right)^{-\frac{1}{3}}$ 9

3. $\frac{(3x^2)^{-1}}{6x^{-3}}$ $\frac{x}{18}$

4. a. -2^4 -16

   b. $(-2)^4$ 16

5. $\frac{3^{-5} \cdot 3^{10}}{3^2}$ 27

6. $\left(4^{-1} + 2^{-1}\right)^2$ $\frac{9}{16}$

   - hint 1: $\left(a^{-m} + a^{-n}\right)^p \quad a^{-mp} + a^{-np}$

   - hint 2: Apply the neg. exponent property to each term. Then get a common denom. and add

V. Logarithms

Given $\log_b a = x$, then $b^x = a$ where $b > 0$ but $b \neq 1$, and $a > 0$.

Directions: - Solve for x.

1. $3\log_2 x = 12$

   $x = 16$

2. $\log_5 125 = x$

   $x = 3$

3. $3 + 4\log_x 4 = 5$

   $x = 16$

4. $\frac{3}{2}\log_{27}(x+5) = 1$

   $x = 4$

5. $1 + \frac{4}{3}\log_{(x-3)} 4 = \frac{11}{3}$

   $x = 5$

6. $\log_{\sqrt{5}} 25^{4x-1} = 3$

   $x = \frac{7}{16}$

VI. Rational Expressions

Directions - Simplify to a single fraction:

1. Hint: get a common denominator!

$$\frac{1}{ab} - \frac{2}{b^2}$$

$$\frac{b-2a}{ab^2}$$

2. Hint: factor and cancel!

$$\frac{x^2+6x+8}{x^2-4}$$

$$\frac{x+4}{x-2}$$

3. Hint: get a common denominator in the numerator and multiply by the reciprocal

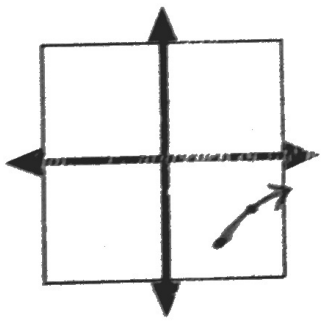
$$\frac{\frac{x}{x-1}+1}{\frac{x+2}{x}}$$

$$\frac{2x^2-x}{x^2+x-2}$$

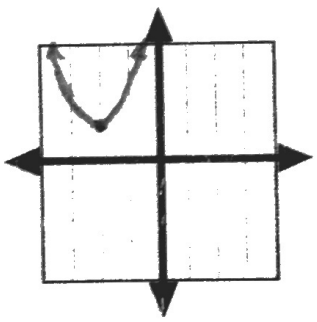
VII. Quick Graphs:

Directions - Graph each of the following.
- If you don't remember, use your graphing calculator to help you determine the patterns. But you need to be able to do these graphs without your calculator!

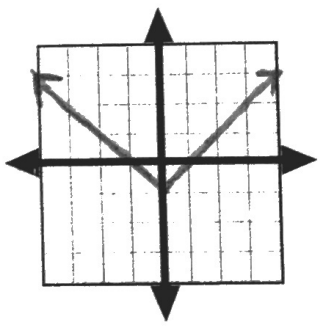
1. $y = \sqrt{x-2} - 3$



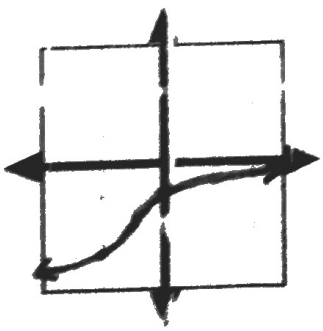
2. $y = (x+2)^2 + 1$



3. $y = |x| - 1$



4. $y = \sqrt[3]{x+1} - 2$



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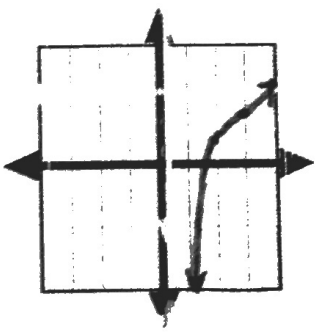
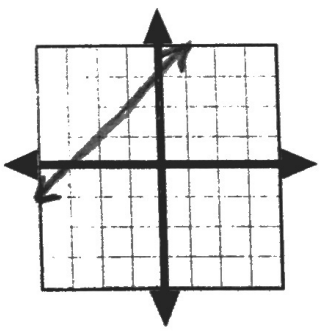


6. $y = x + 3$



VII. Probability - Calculator Allowed - *Round all probabilities to three decimal places.*

1. **Prom Plans** - A group of 10 girls and 6 boys from the sophomore class board are in charge of the 2005 prom. In how many ways can they elect a President, Vice-President and Treasurer?

   3450

2. **Drawing Cards** - Three cards are drawn *all at once* from a standard deck of 52 playing cards. What is the probability of selecting exactly two aces?

   approximately 0.013

3. **Skittles in a Jar** - Stacey has a jar that contains 21 green skittles and 30 yellow skittles. She withdraws one skittle and notes the color. She does not return it to the jar. Stacey then withdraws another skittle. Find the probability that the 1st skittle is green and the next skittle is yellow.

   $\frac{21}{85}$

4. **Trains** - A passenger train consists of 2 different locomotives, 6 different passenger cars and 2 different sleeper cars. In how many ways can the cars be arranged if the locomotives must be together and must be at the beginning of the train?

   80,640

VIII. Regression Equations - Calculator Allowed

Regression equations are equations of lines which are created to provide functions which "best fit" a set of data points. The calculation of such equations is left up to our calculator.

In this problem, you will be asked to create both a quadratic and a linear regression equation for the data provided below. On the following page you will find directions on how to create a quadratic regression equation on the TI-83 or TI-83 Plus. Should you have a different calculator, you will have to look up this topic in the manual.

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t	0	1	2	3	4
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******Round all answers to three decimal places!!!******

1. Using the calculator, create a quadratic regression equation for the data.
   Quadratic Regression Equation: $y = 0.986x^2 + 8.08x + 0.651$

2. Using the calculator, create a linear regression equation for the data.
   Linear Regression Equation: $y = 12.02x - 1.32$

3. Which equation best represents the data? Why do you think that?

   Answers will vary.

4. Using the quadratic model for the data, what was the speed at $t = 1.6$ seconds?

   17.9 m/s

5. Using the quadratic model for the data, what was the speed at $t = 6$ seconds?

   84.6 m/s

6. Using the linear model for the data, when was the speed exactly 23 m/s?

   approximately 2.02 seconds